Antarctic Ice Shelves and Ice Sheets near the Surface

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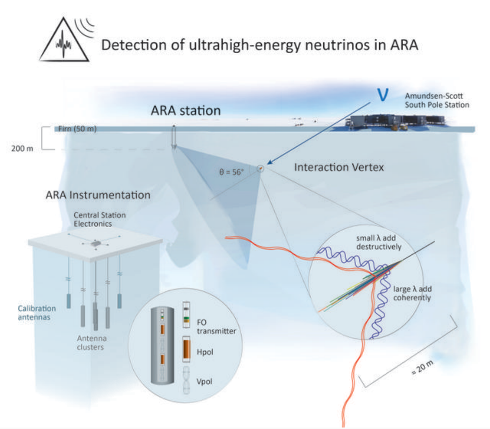
CCAPP @ OSU

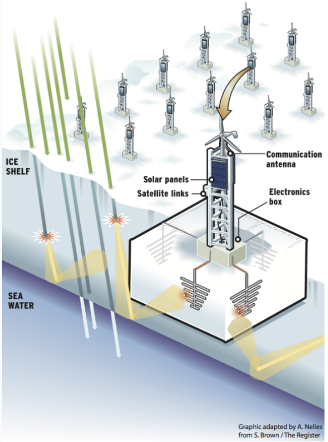
Ten Minute Talk on Ice Shelves and Ice Sheets Near Surface

I. Dr. Connolly has returned from Weizmann Institute with good news
II. Part of those discussions require a review of knowledge for ice studies
    A. Emphasis on firn properties
    B. Emphasis on (possible) surface waves
III. Physics of an ice column
IV. The Landau-Lifshitz/Looyenga equation
V. Review of Results from Moore's Bay (2011-12)
VI. Review of Results from ARA Analyses (2011-present)

Ten Minute Talk on Ice Shelves and Ice Sheets Near Surface

ARIANNA/ARA have a few key differences, that inform how the projects can be combined





Ten Minute Talk on Ice Shelves and Ice Sheets Near Surface

I. Why do we believe that the firn density profile is exponential?

II. Why do we believe index of refraction tracks density?

Theoretical Exponential Density Requirements

Let the firn be represented by N discrete blocks of firn labelled by j, each with a volume $A\Delta z$, and a density ρ_j. A height of 0 m corresponds to the first block ($j = 0$) at the bottom of the stack, and the height of the stack is $h = N\Delta z$. The force down on block j is balanced by a normal force:

$$F_j = \rho_j v_j g + \sum_{i=j+1}^{N} \rho_i v_i g \tag{1}$$

The force down on block $j + 1$ is balanced by a normal force:

$$F_{j+1} = \rho_{j+1} v_{j+1} g + \sum_{i=j+2}^{N} \rho_i v_i g \tag{2}$$

Theoretical Exponential Density Requirements

Using $v_j = v_{+1} = A\Delta z$, we have

$$\frac{F_{j+1} - F_j}{gA\Delta z^2} = \frac{\rho_{j+1} - \rho_j}{\Delta z} + \frac{1}{\Delta z^2}\left(\Delta z \sum_{i=j+2}^{N} \rho_i - \Delta z \sum_{i=j+1}^{N} \rho_i\right) \tag{3}$$

Letting $\Delta z \to 0$, and $z_0 \leftrightarrow j$, $z_1 \leftrightarrow j+1$, we have

$$\frac{F'}{gv} = \rho'(z) + \frac{1}{dz^2}\left(\int_{z_1}^{h} \rho(z)dz + \int_{z_0}^{h} \rho dz\right) \tag{4}$$

We can also combine the integrals by reversing one, and eliminating h.

Theoretical Exponential Density Requirements

This leaves

$$\frac{F'}{gv} = \rho'(z) + \frac{1}{dz^2}\left(\int_{z_0}^{z_1} \rho(z)dz\right) \tag{5}$$

The derivative of the normal force approaches zero as fast as the volume element. **Thus, the left side is a constant.** Taking the derivative with respect to z of both sides, and rearranging:

$$0 = \rho''(z) + \frac{\rho(z_1) - \rho(z_0)}{dz^2} \tag{6}$$

Recall that $v/dz = A$. Multiplying both sides by v and rearranging yields:

$$\rho''(z) = -\left(\frac{A}{v}\right)\frac{\rho(z_1) - \rho(z_0)}{dz} = -\left(\frac{A}{v}\right)\rho'(z) \tag{7}$$

Theoretical Exponential Density Requirements

Finally:

$$\rho''(z) = -\left(\frac{A}{v}\right)\rho'(z) \tag{8}$$

which has the solution ($A/v \equiv z_0^{-1}$):

$$\boxed{\rho(z) = R_0 \exp(-z/z_0) + \rho_0} \tag{9}$$

In this coordinate system, up is positive, $z = 0$ corresponds to the highest density, and $z = h$ is the surface. (The assumptions are not always valid, so be careful). The two remaining constants are derived from the snow density and the bulk ice density.

Landau-Lifshitz/Looyenga Equation

For a material comprised of a mixture of two dielectrics, the complex dielectric constant is:

$$\epsilon_{mix} = \left(v_1\epsilon_1^{1/3} + v_2\epsilon_2^{1/3}\right)^3 \tag{10}$$

The constants v_1 and v_2 are the volume fractions, with $v_1 + v_2 = 1$, $\rho_{mix} = v_1\rho_1 + v_2\rho_2$, and all ϵ are complex. Let the *loss tangent* be defined by $\tan\delta = \epsilon''/\epsilon'$, with $\Re\epsilon = \epsilon'$ and $\Im\epsilon = \epsilon''$. Let us treat the firn as a mixture of ice and snow. Both ice and snow are lossy, but with small loss tangents at RF frequencies ($\tan\delta \leq 10^{-3}$). We can show that

$$\epsilon_i^{1/3} \approx |\epsilon_i|^{1/3}\left(1 + \frac{i}{3}\tan\delta_i\right) \tag{11}$$

Landau-Lifshitz/Looyenga Equation

Using this approximation, to first order in $\tan\delta_i$, we have:

$$\Re\epsilon_{mix} \approx R^3 \tag{12}$$

$$R = v_1|\epsilon_1|^{1/3} + v_2|\epsilon_2|^{1/3} \tag{13}$$

Note that:

$$|\epsilon_i| \approx \epsilon_i' \left(1 + \frac{1}{2}(\tan\delta_i)^2\right) \approx \epsilon_i' \tag{14}$$

The (real) index of refraction is

$$n \equiv \sqrt{\Re\epsilon_{mix}} = \left(v_1^3\epsilon_1' + v_2^3\epsilon_2' + 3(v_1v_2^2\epsilon_1'\epsilon_2' + v_1^2v_2\epsilon_1'\epsilon_2')\right)^{1/2} \tag{15}$$

$$n = v_1^{3/2}\left(\epsilon_1' + u^3\epsilon_2' + 3u(u+1)\epsilon_1'\epsilon_2'\right)^{1/2} \tag{16}$$

$$u = v_2/v_1 \tag{17}$$

Taking the example $u = 0$, $v_1 = 1$ as a check yields $n = \sqrt{\epsilon_1'}$, the result for a uniform linear dielectric.

Landau-Lifshitz/Looyenga Equation

$$n = v_1^{3/2}\left(\epsilon_1' + u^3\epsilon_2' + 3u(u+1)\epsilon_1'\epsilon_2'\right)^{1/2} \tag{18}$$

From the definition of the v_i:

$$\left(\frac{\rho_{mix}}{\rho_1}\right)^{3/2} \propto v_1^{3/2} \tag{19}$$

This implies

$$n \propto \rho_{mix}^{3/2} \tag{20}$$

Landau-Lifshitz/Looyenga Equation and Density Profile

Now we are ready to conclude. The density profile is exponential:

$$\rho_{mix} \propto \exp(-z/z_0) \tag{21}$$

and the index of refraction is like a power-law of the density:

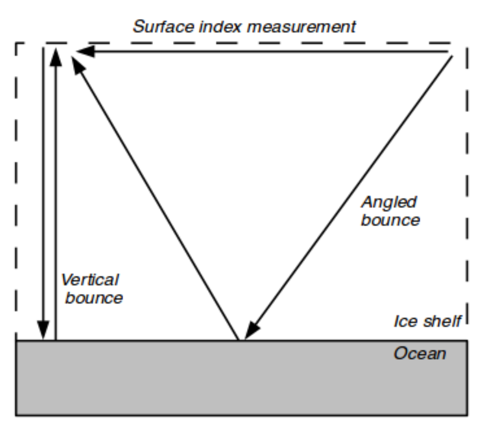
$$n \propto \rho_{mix}^{3/2} \tag{22}$$

Because raising an exponential function to some power yields another exponential:

$$\boxed{n \propto \exp(-n/n_0)} \tag{23}$$

Why do we expect an exponential index of refraction versus depth? Because of 1) gravity and Newton's second law, and 2) dielectric mixing.

Review of Results from Moore's Bay in 2011-12 Season



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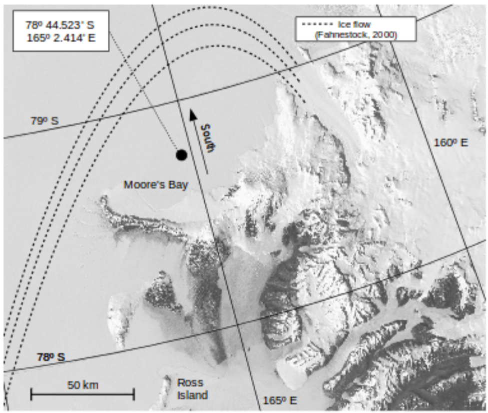

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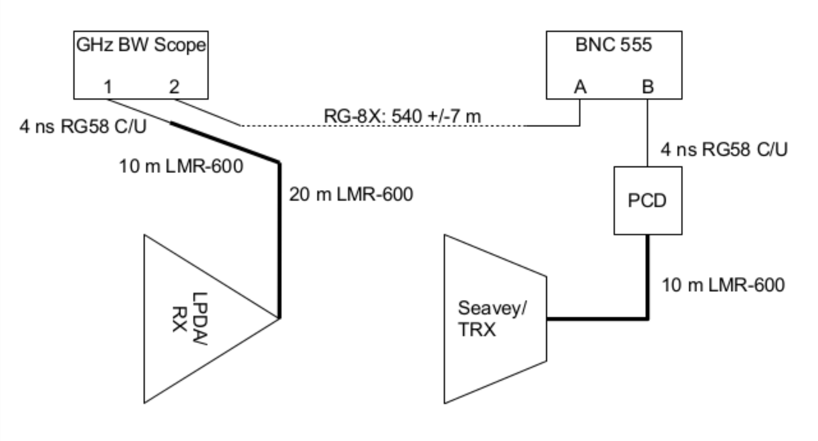

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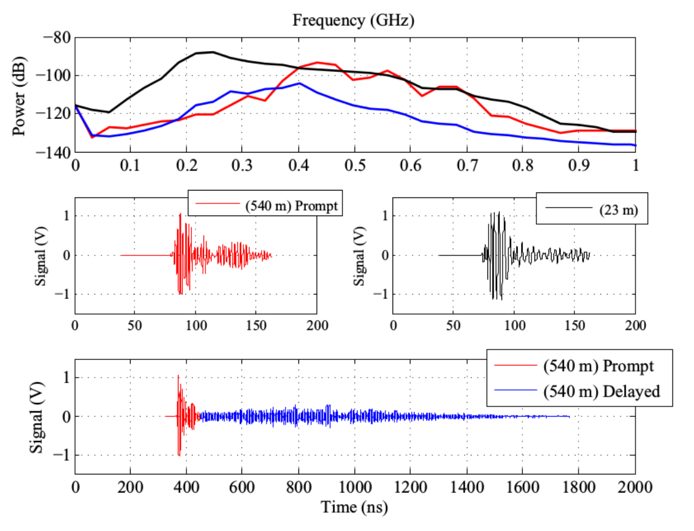
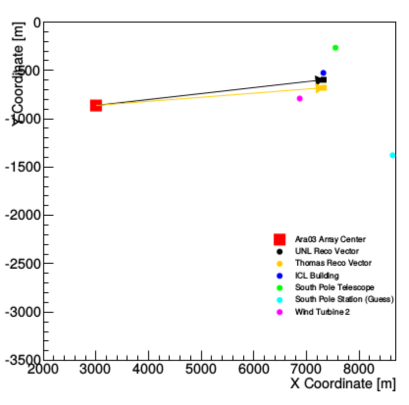


Review of Results from Moore's Bay in 2011-12 Season

Year	Δt_{meas}	Δt_{phys}	σ_{stat}	σ_{sys}	σ_{pulse}	σ_{tot}	d_{ice} (m)
2006	-	6783	-	-	-	10	577.5 ± 10
2009	-	6745	-	-	-	15	572 ± 6
2010	7060	6772	5.0	8.0	10	14	576 ± 6
2011	6964	6816	4.0	5.0	10	12	580 ± 6

Review of Results from South Pole Station

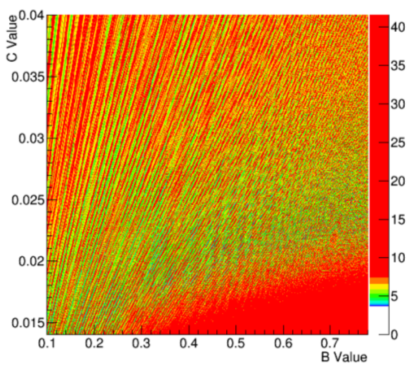
Result from Kravchenko et. al. (2015) builds on prior analyses of surface pulser data by Thomas and others. The pulser is at the South Pole Station, 4 km away, and the target is the ARA03 array center. Ray tracing is used to predict the relative timing offsets between antennas and strings.



Review of Results from South Pole Station

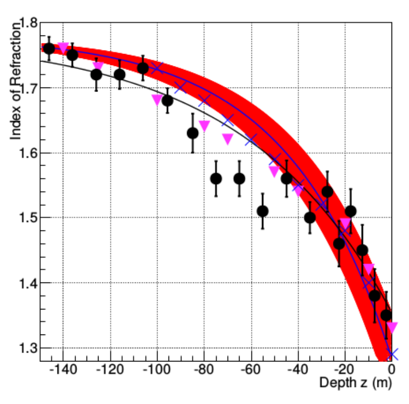
The following function (now well-motivated) is fit to the data, assuming the asymptotic value of $n = 1.78$ for bulk ice at large depths (here increasing z means deeper).

$$n(z) = A - B\exp(-Cz) \qquad (24)$$



Review of Results from South Pole Station

Result from Ilya Kravchenko and others (2015) builds on prior analyses of surface pulser data (Gow with $n = 1 + k\rho(z)$, Eisen Maud Dronning Core, Schytt theoretical models, RICE data. **My question:** This model requires the *assumption* of some model of $n(z)$, and then uses ray-tracing to predict *relative* time-differences between channels. If ray-tracing predicts a different incoming direction than a straight line path, what is that difference in angle?



Review of Results from South Pole Station

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Radio surf in polar ice: A new method of ultrahigh energy neutrino detection

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(Received 6 October 2004; published 24 January 2005)

We explore a new mechanism for detection of neutrino-induced showers via surface waves at radio frequencies. Air-dielectric surface waves exist on a plane boundaries that have attenuation lengths 2.82 that of the dielectric, and their amplitudes fall only by the inverse square root of propagation distance. Allowing for substantial coupling uncertainties, surface waves may provide a useful mechanism for neutrino detection in polar ice, promoting the development of neutrino telescopes in the energy regime above 10^{15} eV.

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VII. **Surface Waves**