HIGH-ENERGY PHYSICS AND RADIOGLACIOLOGY

J. C. Hanson (CCAPP, The Ohio State University)

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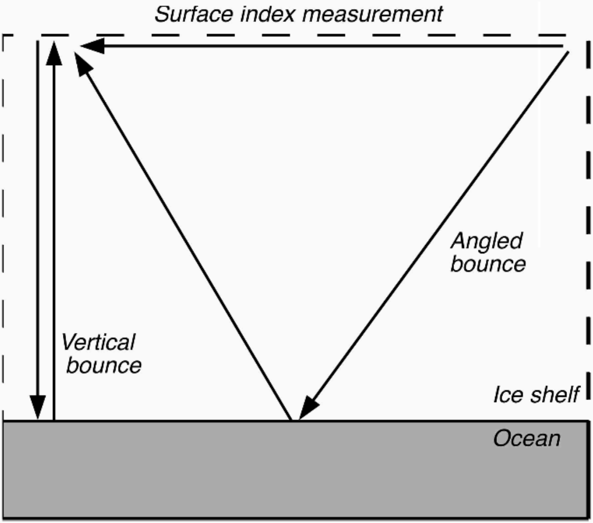
Colloquium for the Department of Physics and Astronomy, Whittier College

I. Radioglaciology
    A. Using radio waves (< 1000 MHz) to probe ice properties
    B. RF properties of ice
        i. Attenuation (absorption, scattering)
        ii. Reflections
        iii. Index of refraction profile
II. Specific Results found as part of the ARIANNA program
    A. Thickness
    B. Attenuation
    C. Reflection
III. Surface Propagation


Surface index measurement
Angled bounce
Vertical bounce
Ice shelf
Ocean

RF PROPERTIES OF ICE

I. *Index of refraction, n*: $v = c/n$, usually take $n = n' + in''$
II. *DIelectric constant, ϵ*: $\epsilon = \epsilon' + i\epsilon''$
III. $n = \sqrt{\epsilon}$
IV. Propagating E-field in free space: $\mathsf{E} = \mathsf{E}_0 \exp\left(i(kz - \omega t)\right)$
V. The *wavevector* is $k = 2\pi\nu/c$ in free space
VI. Propagating E-field in dielectric medium:
$\mathsf{E} = \mathsf{E}_0 \exp\left(i(nkz - \omega t)\right)$
VII. The *wavevector* is $k = 2n\pi\nu/c$ in dielectric medium

$$\epsilon = \epsilon' + i\epsilon'' \tag{1}$$

$$n \equiv \sqrt{\epsilon} = (\epsilon' + i\epsilon'')^{1/2} \tag{2}$$

$$n \approx \sqrt{\epsilon'}(1 + i/2 \tan\delta) \tag{3}$$

$$n'' = \Im\{n\} \approx \frac{1}{2}\sqrt{\epsilon'}\tan\delta \tag{4}$$

$$k = \frac{2\pi\nu}{c} \tag{5}$$

$$L^{-1} = n''k = \frac{\pi}{c}\sqrt{\epsilon'}(\nu\tan\delta) \tag{6}$$

$$N_L(dBkm^{-1}) = 8686.0L^{-1} \tag{7}$$

$$\epsilon(\omega) = \epsilon_\infty + \frac{\Delta\epsilon}{1 + i\omega\tau} \tag{8}$$

$$\Delta\epsilon = \epsilon_s - \epsilon_\infty \tag{9}$$

$$\tan\delta = \frac{\Delta\epsilon(\omega\tau)}{\epsilon_s + \epsilon_\infty(\omega\tau)^2} \tag{10}$$

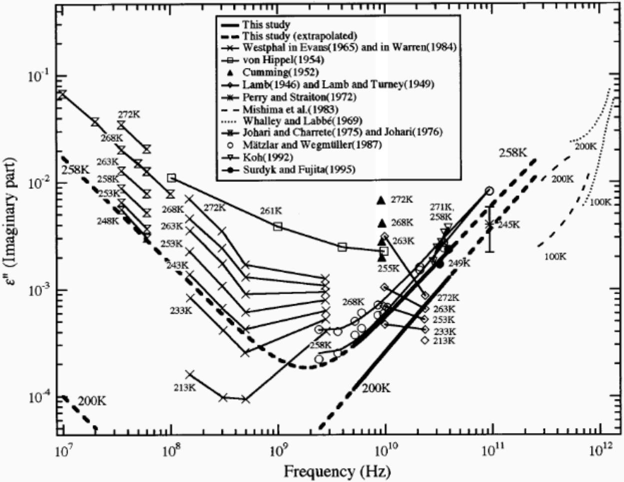
$$\tag{11}$$

For high-frequencies such that ($\omega\tau \gg 1$)

$$\tan\delta \approx \frac{\Delta\epsilon}{\epsilon_\infty}(\omega\tau)^{-1} \tag{12}$$

$$\nu \tan\delta \propto \omega \tan\delta = const \tag{13}$$

RF PROPERTIES OF ICE - MATSUOKA, FUJITA, MAE (1996)



$$\tan\delta \approx \frac{\Delta\epsilon}{\epsilon_\infty}(\omega\tau)^{-1} \tag{14}$$

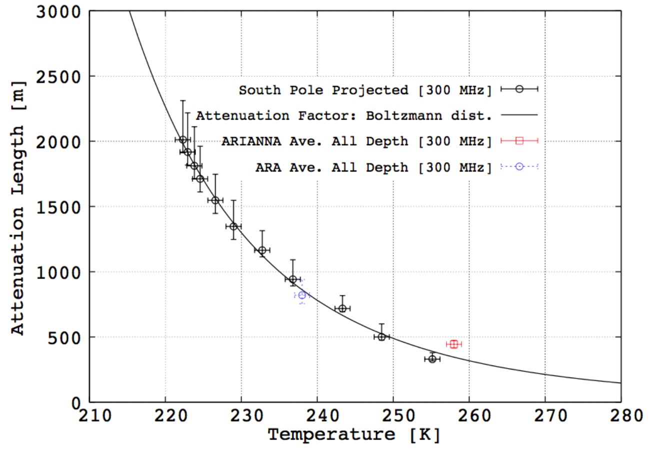
But the Debye relaxation time τ is inversely proprotional to a molecular transition rate, which depends on temperature via the Boltzmann distribution:

$$\tau = A\exp(E_a/k_B T) \tag{15}$$

Thus, attenuation length *depends on ice temperature*:

$$L \approx \frac{2cA}{\pi n}\frac{\epsilon_\infty}{\Delta\epsilon}e^{E_a/k_B T} \tag{16}$$

RF PROPERTIES OF ICE - $\nu \tan \delta$ AND THE RELAXATION TIME



Reflections occur when there are two different indexes of refraction:

$$|\sqrt{R}| = \frac{1 - n_2/n_1}{1 + n_2/n_1} \tag{17}$$

Let $\alpha = \epsilon_2''/\epsilon_1'$, $\tan\delta_2 \gg 1$, and $\tan\delta_1 \approx 0$. This yields:

$$|\sqrt{R}| \approx \left(\frac{1 + \alpha - \sqrt{2\alpha}}{1 + \alpha + \sqrt{2\alpha}}\right)^{1/2} \tag{18}$$

Thus, for a situation like ice over the ocean, $|\sqrt{R}| \sim 0.4 - 1$.

To solve for the ice thickness in terms of the reflection time:

$$\frac{c\Delta t}{2} = \int_0^{d_{ice}} n(z)dz \tag{19}$$

The Schytt model:

$$n(z) = 1.78 \quad z \geq D_f \tag{20}$$

$$n(z) = n_{ice} - \Delta n \exp(z/z_0) \quad z < D_f \tag{21}$$

$$\Delta n = n_s - n_{ice} \tag{22}$$

Knowing d_{ice} independently allows determination of attenuation length and reflection coefficient.

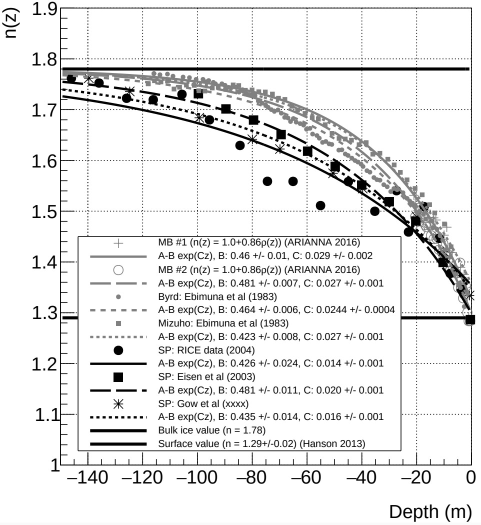
$$V_C(\nu) = V_0/d_c \tag{23}$$

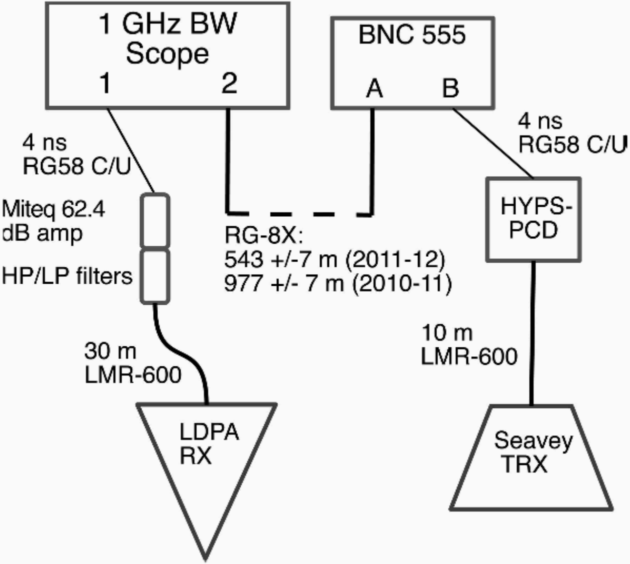
$$V_{ice}(\nu) = \sqrt{R}\frac{V_0}{d_{ice}} \exp\left(-\frac{d_{ice}}{L(\nu)}\right) \tag{24}$$

$$L(\nu) = \frac{d_{ice}}{\ln((V_C(\nu)/d_c)/(\sqrt{R}V_{ice}(\nu)/d_{ice}))} \tag{25}$$

SPECIFIC MEASUREMENTS MADE AS PART OF THE ARIANNA PROGRAM

RF PROPERTIES OF ICE – INDEX OF REFRACTION PROFILE

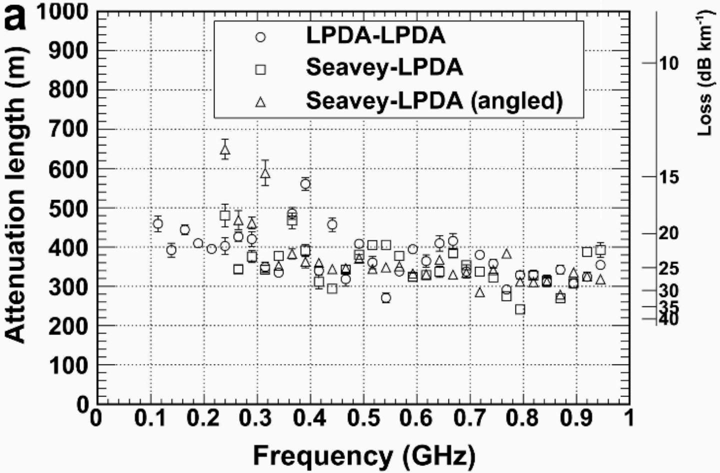



1 GHz BW
Scope
1
2
BNC 555
A
B
4 ns
RG58 C/U
4 ns
RG58 C/U
Miteq 62.4
dB amp
HP/LP filters
RG-8X:
543 +/-7 m (2011-12)
977 +/- 7 m (2010-11)
HYPS-
PCD
10 m
LMR-600
30 m
LMR-600
LDPA
RX
Seavey
TRX

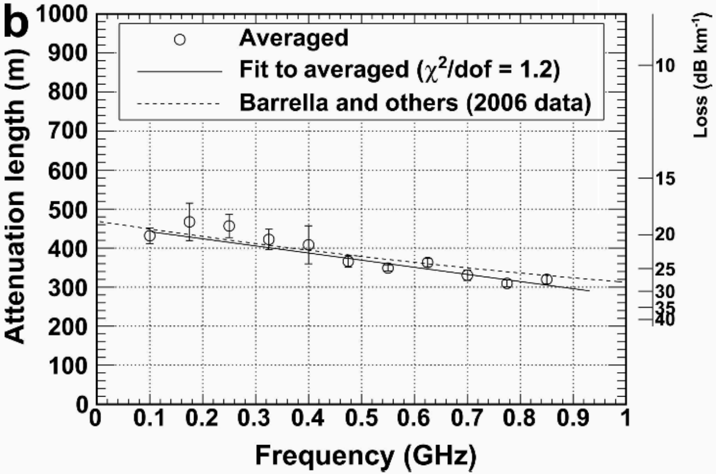
RF PROPERTIES OF ICE – INDEX OF REFRACTION PROFILE YIELDS ICE THICKNESS

Year	Δt_{meas} ns	Δt_{phys} ns	σ_{stat}	σ_{sys}	σ_{pulse}	σ_{tot}	d_{ice} m
2006	–	6783	–	–	–	10	577.5 ± 10
2009	–	6745	–	–	–	15	572 ± 6
2010	7060	6772	5.0	8.0	10	14	576 ± 6
2011	6964	6816	4.0	5.0	10	12	580 ± 6

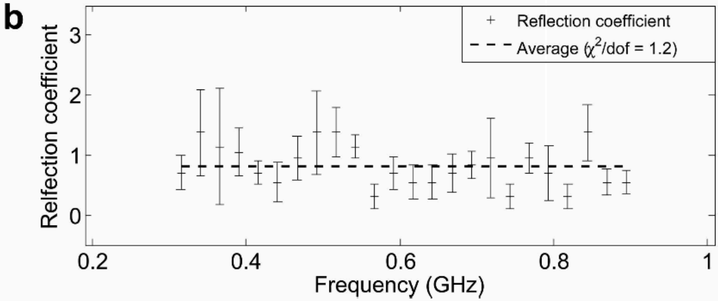
RF PROPERTIES OF ICE - THICKNESS YIELDS ATTENUATION LENGTH (AVERAGE)



RF PROPERTIES OF ICE - THICKNESS YIELDS ATTENUATION LENGTH (AVERAGE)



RF PROPERTIES OF ICE – ATTENUATION LENGTH MEASUREMENTS YIELD $\sqrt{r}$



RF PROPERTIES OF ICE –AVERAGE $\sqrt{r}$ CORRECTS ATTENUATION LENGTHS

ν GHz	$\langle L_0 \rangle$ m	$\langle L \rangle$ m	$\langle L \rangle$ dB km^{-1}	$\epsilon'' \times 10^3$	$\nu \tan\delta \times 10^4$ GHz
0.100	432	449	19.3	3.8	1.2
0.175	467	487	17.8	2.0	1.1
0.250	457	476	18.2	1.4	1.1
0.325	422	438	19.8	1.2	1.2
0.400	408	423	20.5	1.0	1.3
0.475	366	378	23.0	0.95	1.4
0.550	349	360	24.1	0.86	1.5
0.625	363	375	23.2	0.72	1.4
0.700	331	341	25.5	0.71	1.6
0.775	310	319	27.2	0.69	1.7
0.850	320	329	26.4	0.61	1.6
Ave.	380 ± 16	400 ± 18	22 ± 1	1.3 ± 0.3	1.37 ± 0.06

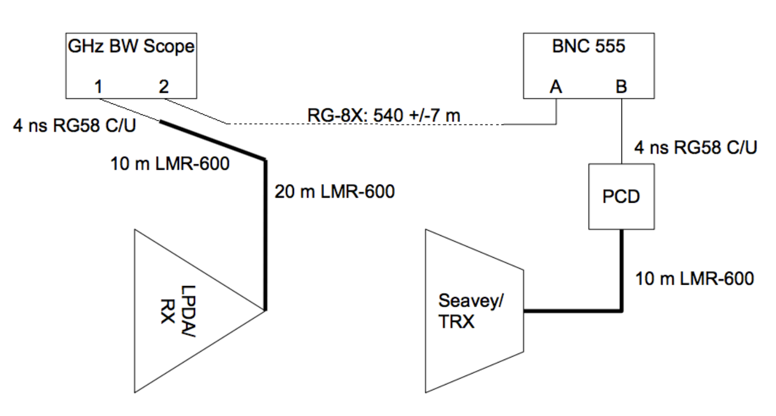
SURFACE PROPAGATION

If RF waves are able to propagate along the firn boundary in two-dimdensions, then $V(\nu) \propto 1/\sqrt{d}$ (roughly). Also, the volume goes like $2\pi r dr$, because thickness isn't changing. Thus, the ratio of these two numbers goes like $\sqrt{r}$, meaning the farthest events are the most prevalent. Also,

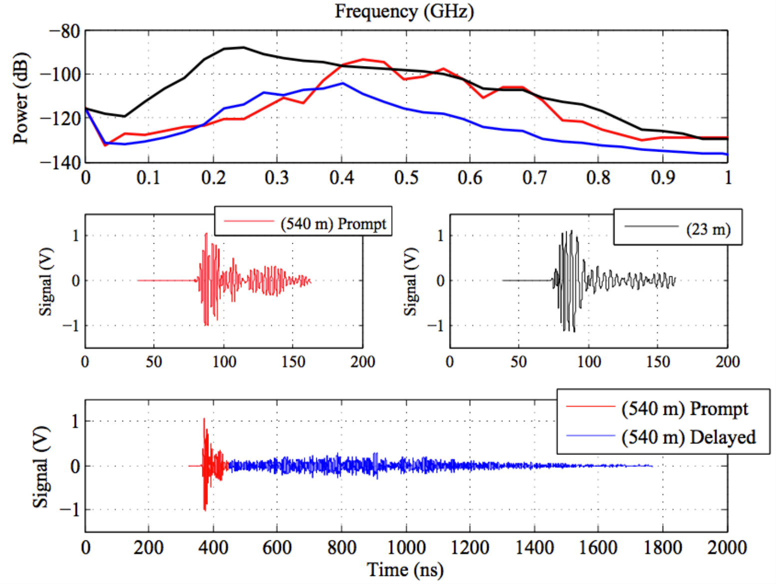
$$\frac{V_{surf}}{V_{bulk}} \approx \left(\frac{\omega r}{c}\right)^{1/2} \tag{26}$$

Surface signals are larger because the lose amplitude less easily, and the effect is stronger for high-frequencies.

SURFACE PROPAGATION – EXPERIMENTAL TEST



SURFACE PROPAGATION – EXPERIMENTAL RESULTS

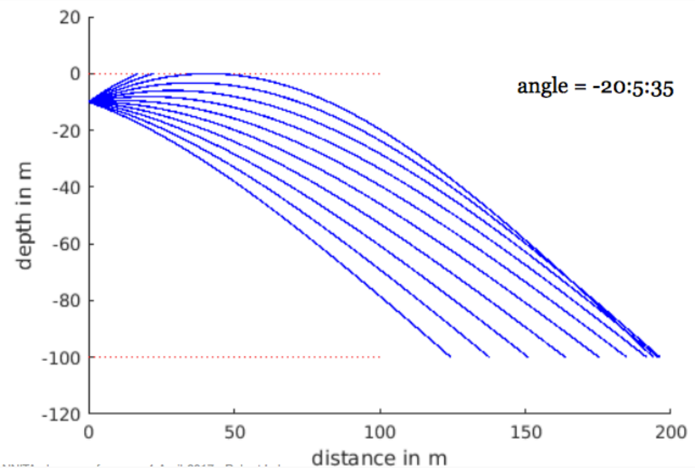


$$n\cos(\alpha) = const \tag{27}$$

$$\frac{d\alpha}{dt} = \cos(\alpha)\frac{c_0}{n(z)^2}\frac{dn}{dz} \tag{28}$$

The first equation is Snell's Law. The angle α is defined with respect to the horizontal. We can implement this in a model, with initial RF propgation conditions, and use our knowledge of $n(z)$.

SURFACE PROPAGATION - EXPERIMENTAL MODELING (ROBERT LAHMANN)



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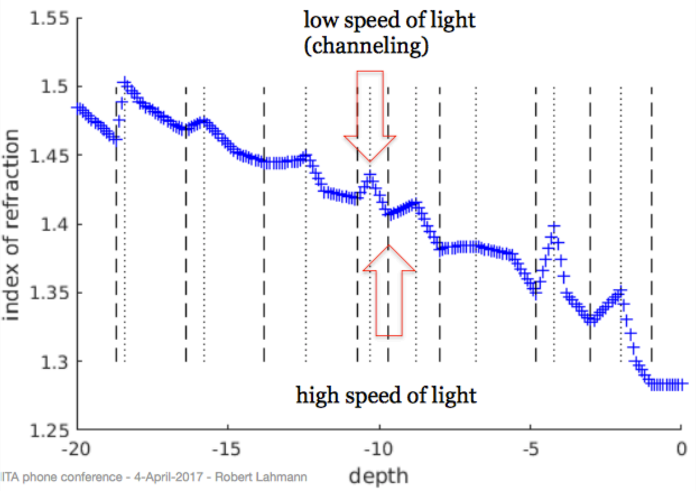

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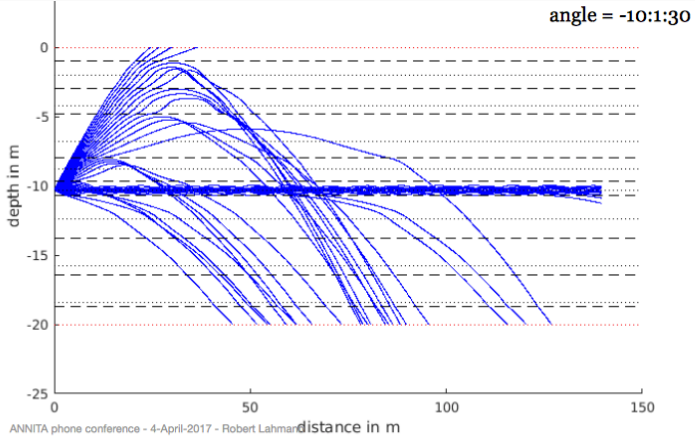

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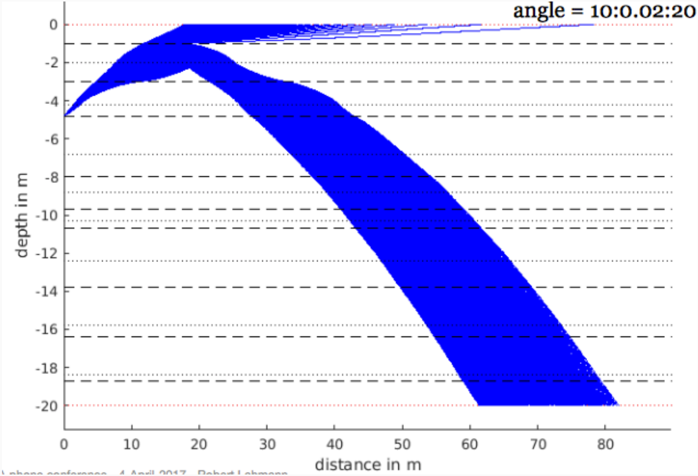

CONCLUSIONS

I. Radio-echo sounding was the tool to do *in-situ* calibration of detection ice
II. Quantified the ice thickness, attenuation length vs. RF, and basal reflection coefficient
III. In agreement with laboratory and field data
IV. Sets the scale of the full detector
V. Journal of Glaciology, Vol. 61 No. 227, 2015
VI. Explanation of surface propagation: RF channeling, no shadowing effect
    A. Other examples of surface propagation being published
        i. Single-frequency, 3km propagation at South Pole
        ii. ARIANNA detector and independent pulser