

Ray tracing in ARA and ARIANNA

SPOORTHI NAGASAMUDRAM
THE OHIO STATE UNIVERSITY

The Askaryan Radio Array (ARA) and The Antarctic Ross Ice Shelf Antenna Neutrino Array (ARIANNA)

- The Askaryan Radio Array (ARA) consists of radio antennas placed 200 m below the surface of ice at the South Pole.
- The Antarctic Ross Ice Shelf Antenna Neutrino Array (ARIANNA) detects neutrinos at the surface that are reflected off of the ice-sea boundary at Moore's bay.
- Both use the Askaryan effect to detect neutrinos.

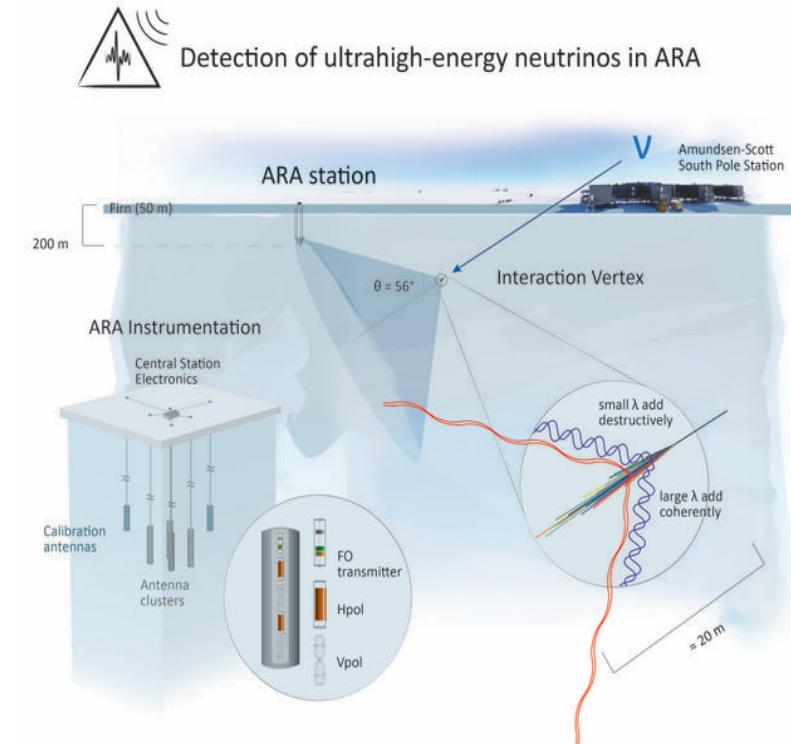
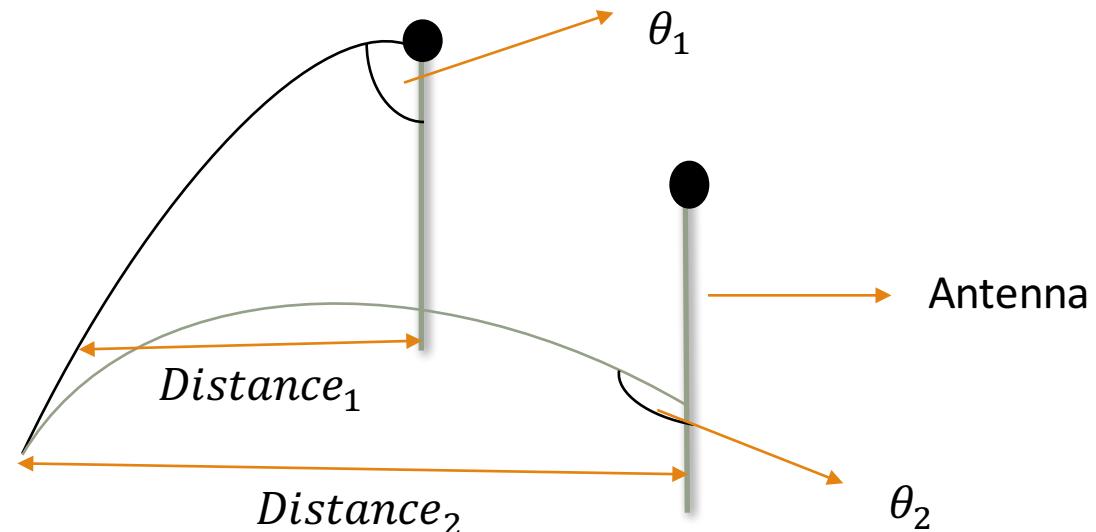


Fig.1. Detection of UHE neutrinos in ARA

Why study ray propagation?

- Determination of optimal position of detectors to capture radio signals.
- Determination of angles at which signals hit the antenna.
Effectiveness of antennas in capturing signals depends on angle at which signals hit the antenna.
- Convenient way to calculate attenuation length at different locations of ice at the South Pole.
- Studying time delays between signals at different antennas which may lead to possible determination of background.



Modeling the index of refraction of ice versus the depth

- Modeling the index of refraction is an important step to solving ray propagation problems.
- Different models based directly on depth and ice density as a function of depth.

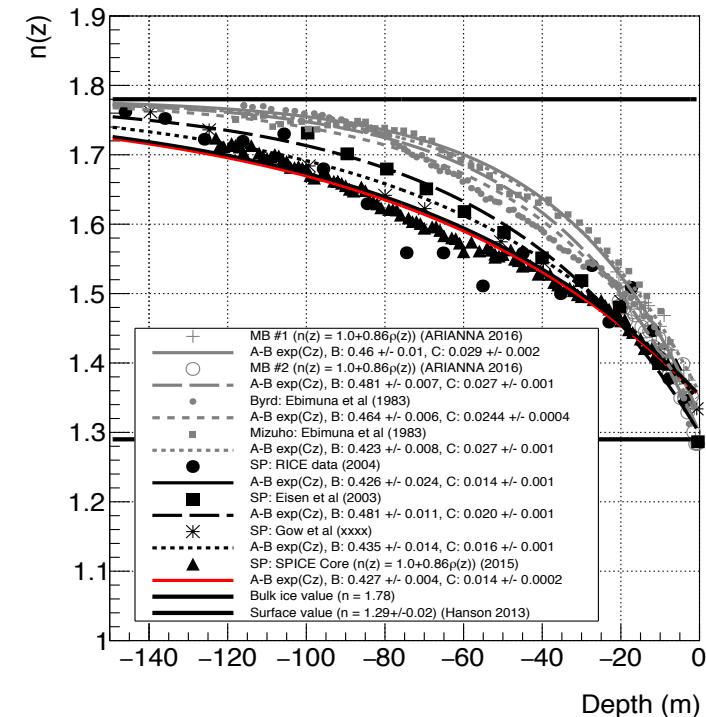


Fig. 2. Fits to the index of refraction of ice vs. its depth

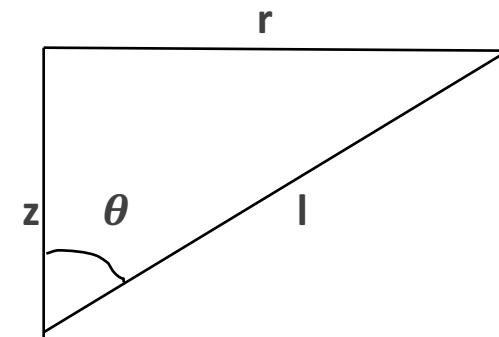
Ray propagation

$$c_0 = nc$$

Assume $\alpha = n \sin \theta$ is constant (Snell's Law)

$$\begin{aligned}\frac{d\alpha}{dl} = 0 &= \frac{dn}{dl} \sin \theta + n \cos \theta \frac{d\theta}{dl} \\ &= \frac{dn}{dz} \sin \theta \cdot \cos \theta + n \cos \theta \frac{d\theta}{dl} \\ &= \cos \theta \left(\frac{dn}{dz} \sin \theta + n \frac{d\theta}{dl} \right)\end{aligned}$$

$$\Rightarrow \frac{d\theta}{dz} = -\tan \theta \cdot \frac{1}{n} \cdot \frac{dn}{dz}$$



Useful relations:

$$dz = \cos \theta \cdot dl = \cos \theta \cdot dt \cdot \frac{c_0}{n}$$

$$dr = \tan \theta \cdot dz$$

Ray propagation

$$1. \frac{d\theta}{dz} = -\tan\theta \cdot \frac{1}{n} \cdot \frac{dn}{dz}$$

$$2. \frac{dr}{dz} = \tan\theta$$

$$3. \frac{dt}{dz} = n \cdot \frac{1}{c_0} \cdot \frac{1}{\cos\theta}$$

ARA ray propagation model results

- Dave Seckel (ARA): ray tracing by explicit solutions of differential equations.

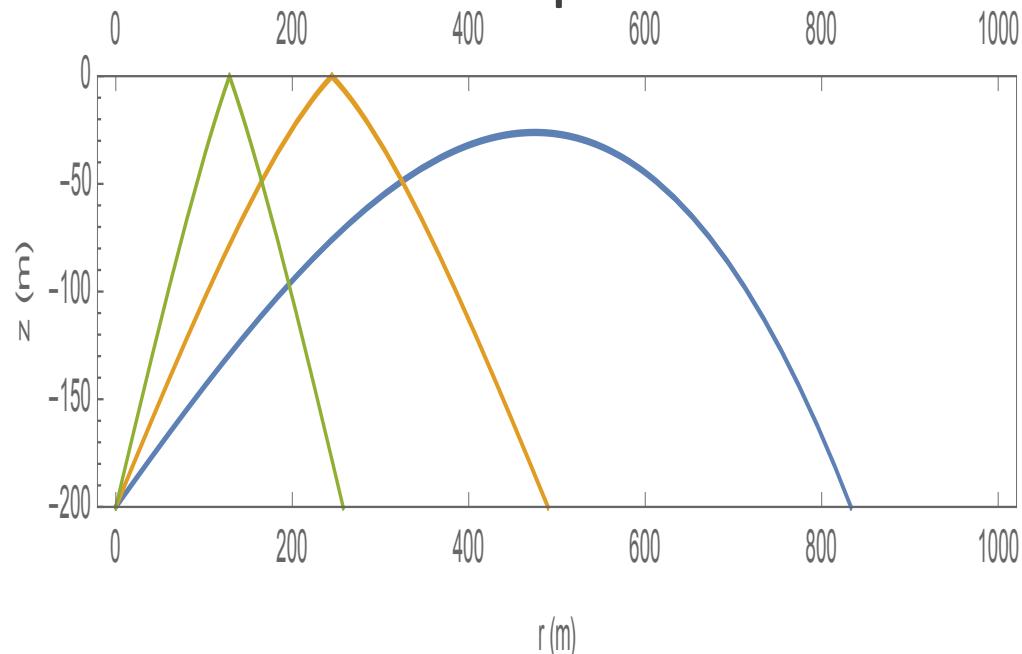


Fig. 3. Modeling of rays using Mathematica

Index of refraction of ice model used:

$$n(z) = 1.78 - 0.43e^{0.0132z}$$

ARIANNA ray propagation model results

- Robert Lahmann (ARIANNA)

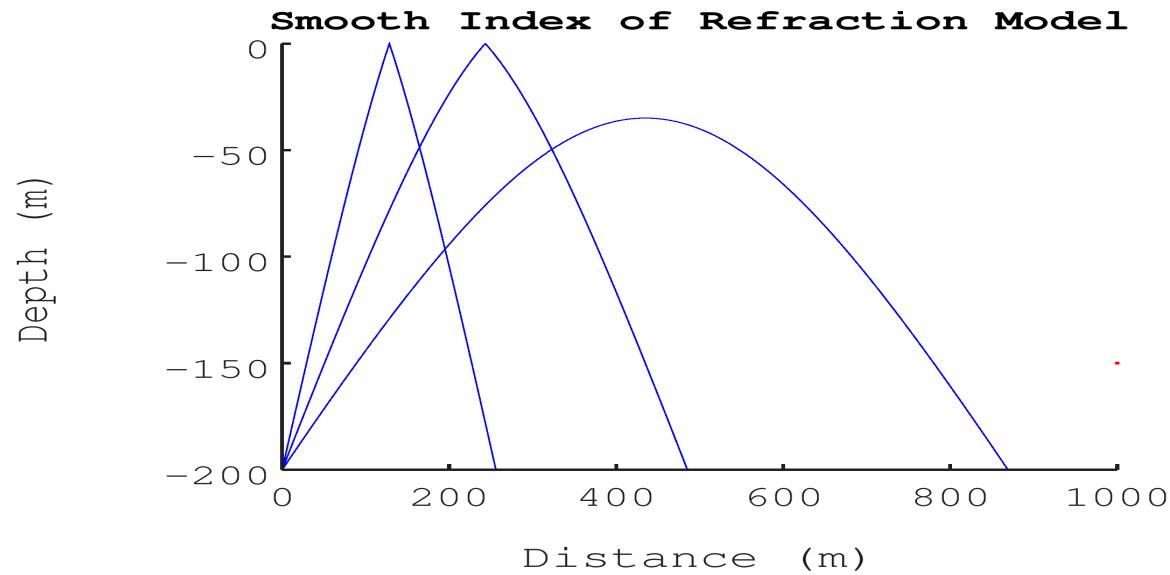


Fig. 4. Modelling of rays using Octave

Problems?

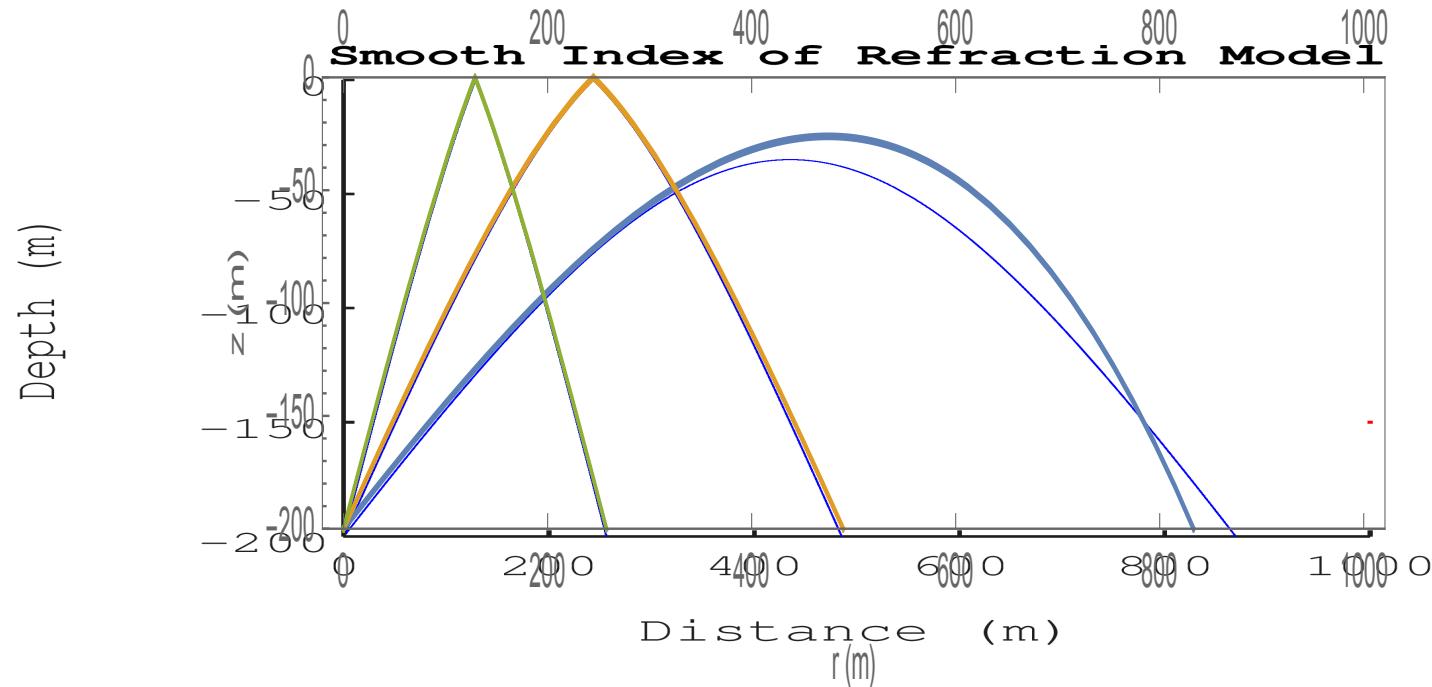


Fig. 5. Overlay of Mathematica and
Octave graphs

Conclusion/further work

- Ray propagation is important in
 - determining the optimal position of detectors to capture radio signals.
 - angles at which signals hit the antenna which further help to improve the efficacy of antennas capturing radio signals.
 - calculating attenuation length at different ice locations in Antarctica.
 - studying time delays between signals at different antennas which may lead to possible determination of background.
- Further work in understanding the inconsistency in both the Mathematica and Octave code
- Be able to incorporate most appropriate ray propagation model into AraSim.