Ray Tracing in Antarctic Ice

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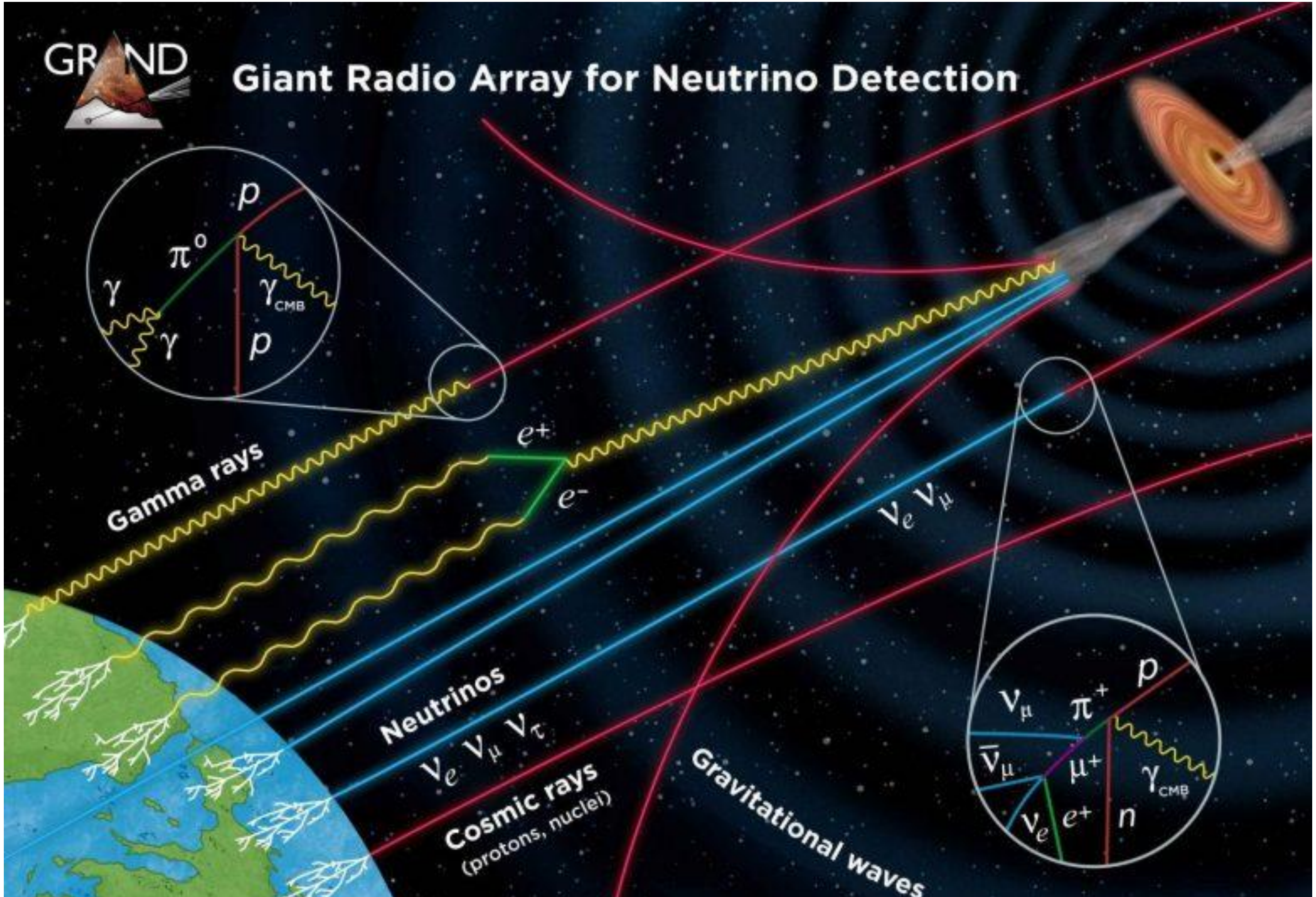
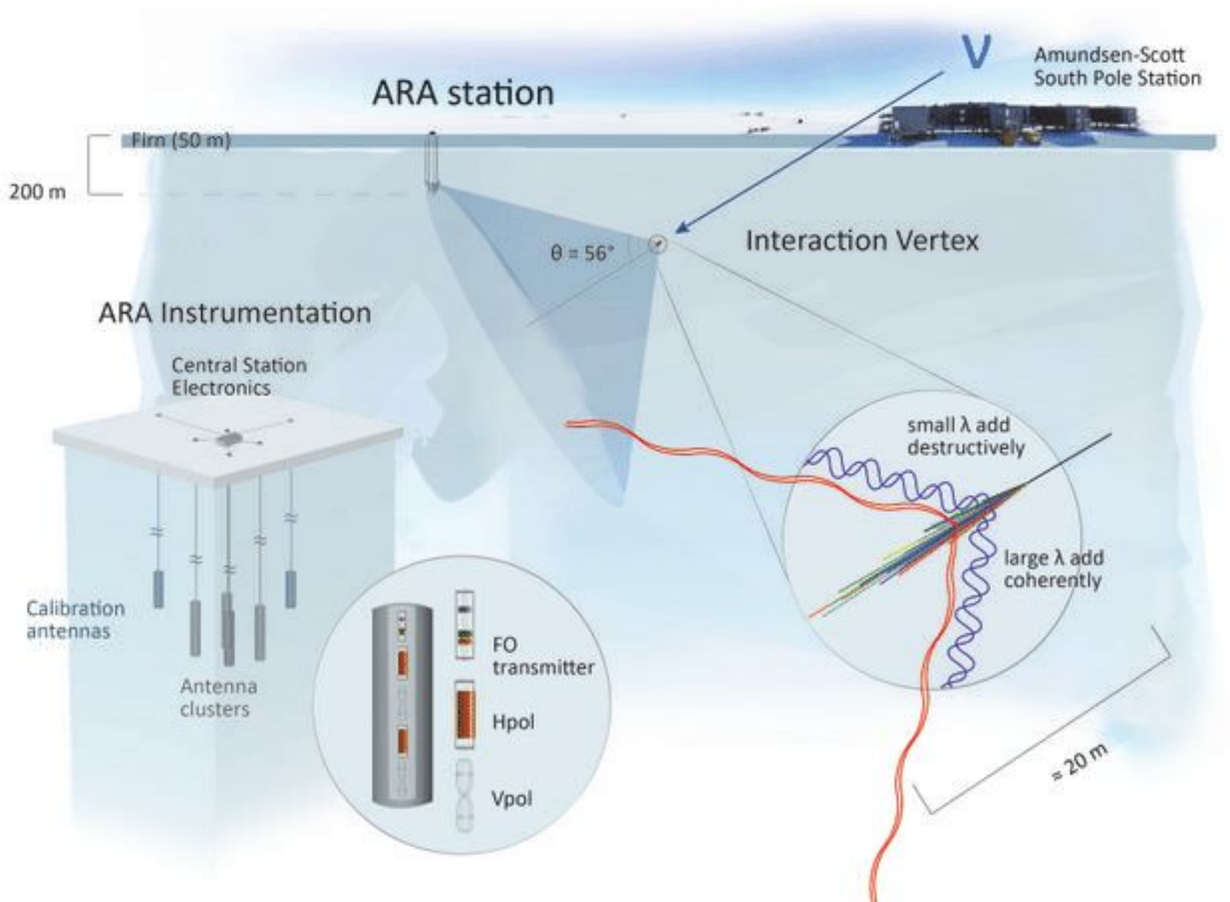
Astrophysics



Detection of ultrahigh-energy neutrinos in ARA



Ray Trace Importance

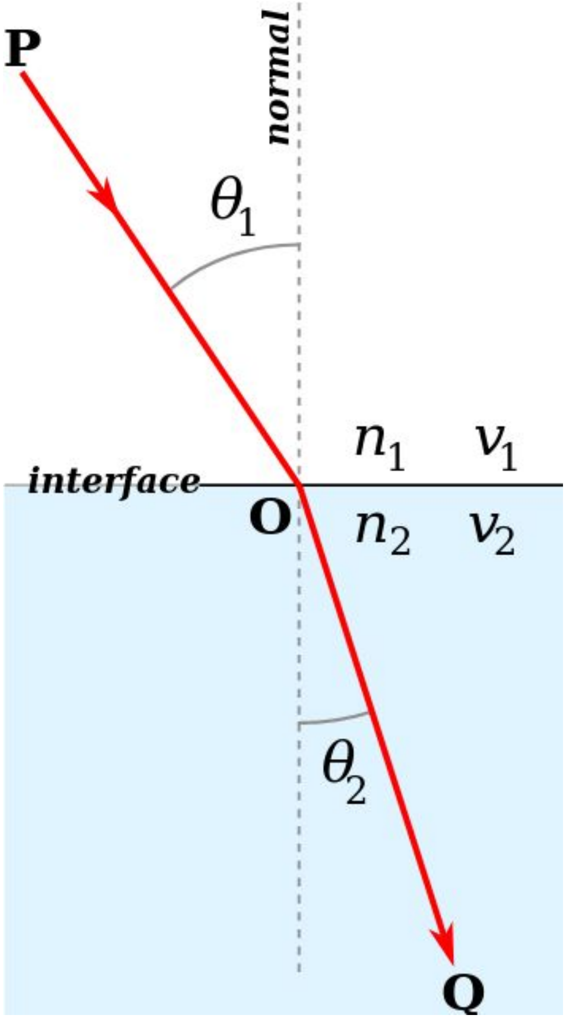
- Reconstruct interaction vertex
  - Accuracy, angular resolution, etc.
- Index refraction changes
  - Ice density related to depth

$$n(z) = A + Be^{Cz}$$

A=1.78, B=-0.43, C=0.0132 (m^{-1})

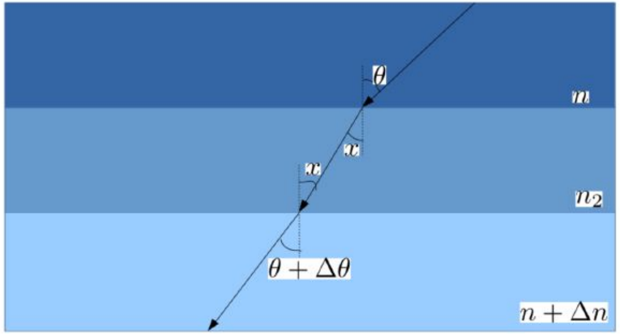
Optical

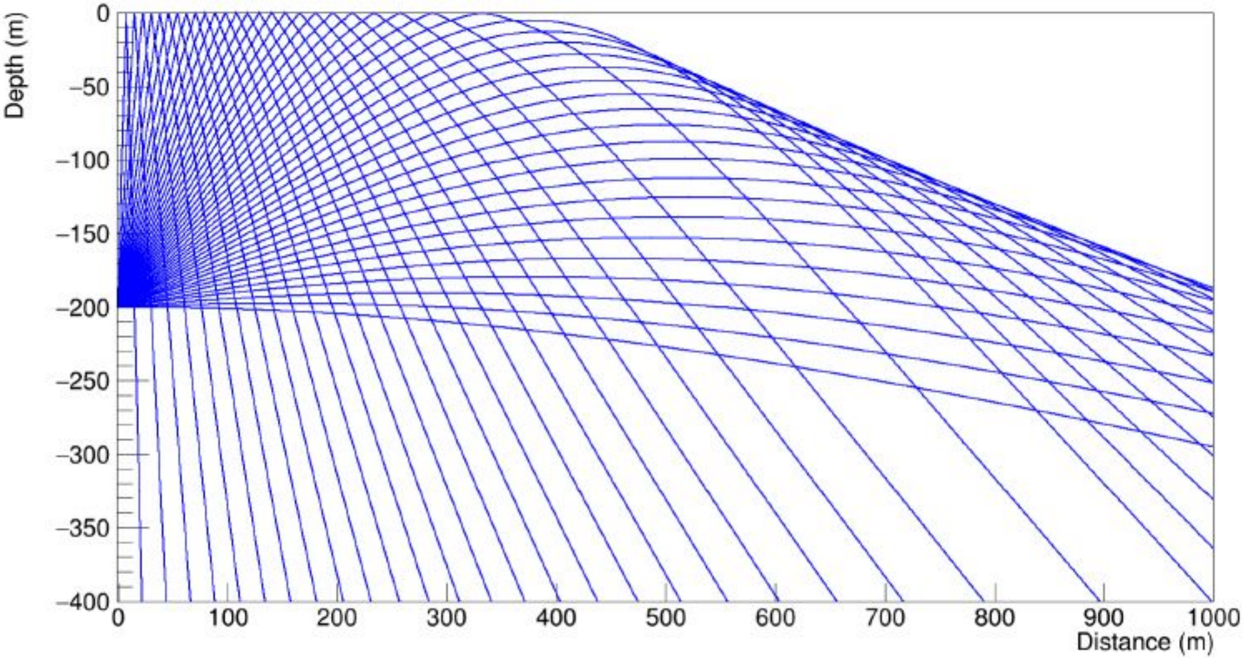
Material	Index of Refraction (n)
Vacuum	1.000
Air	1.000277
Water	1.333333
Ice	1.31
Glass	About 1.5
Diamond	2.417



Shadow Zone and Ray Bending

- Rays
  - Direct
  - Reflected
  - Refracted

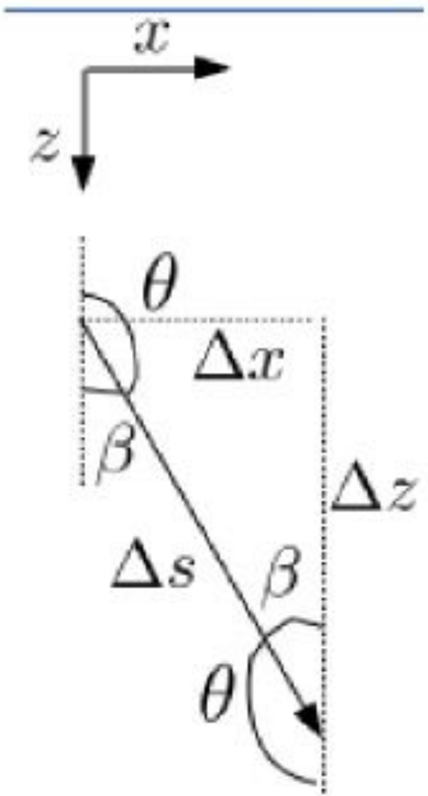




Numerical Approach

- Runge-Kutte Method
  - Brute Force will always work
  - Parmaters
    - A_0 initial signal amplitude
    - L(z,f) attenuation length
    - Beta is the angle

- Probem
  - 1 mm increments
  - ~ km scale
  - Computation time!



$$\frac{dx}{ds} = \sin(\beta)\ , \quad \frac{dz}{ds} = \cos(\beta)\ , \quad \frac{dt}{ds} = \frac{n(z)}{c}$$

$$\frac{d\beta}{ds} = -\frac{\sin(\beta)}{n(z)}\frac{dn(z)}{dz}\bigg|_z\ , \quad \frac{dA_0}{ds} = \frac{A_0}{L(z,f)}$$

Partial Analytic Solution

$$n \sin(\theta) = n_2 \sin(x), \quad n_2 \sin(x) = (n + \Delta n) \sin(\theta + \Delta\theta).$$

$$\frac{\sin(\theta + \Delta\theta)}{\sin(\theta)} = \frac{n}{n + \Delta n}.$$

$$\boxed{\frac{d\theta}{dn} = -\frac{\tan(\theta)}{n(z)}}$$

$$L = n(z) \sin(\theta)$$

$$\boxed{L = n(z_0)\sin(\theta_0)}$$

$$\theta = \arcsin\left(\frac{L}{n(z)}\right)$$

$$\frac{dx}{dz} = \tan(\theta) = \tan\left(\arcsin\left(\frac{L}{A + Be^{Cz}}\right)\right)$$

$$x(L,z) = \frac{L}{C}\frac{1}{\sqrt{A^2 - L^2}}\left(Cz - \log\left(A(A + Be^{Cz}) - L^2 + \sqrt{A^2 - L^2}\sqrt{(A + Be^{Cz})^2 - L^2}\right)\right)$$

Crazy looking equation but very important properties!

- Function undefined when A = L
  - Limits angles to be less 90 degrees
- Function undefined at n(z) = L
  - Gives us max height for z
  - This is the turning point where the ray will bend back down

$$z_{max} = \frac{1}{C}\log\left(\frac{L - A}{B}\right)$$

Finding Launch Angle

- Given
  - (x_0, z_0) & (x_1, z_1)

1. For Direct rays: $f_1'(L,z) = f_1(L,z) - f_1(L,z_0)$.
2. For Reflected and Refracted rays: $f_2'(L,z) = f_2(L,z) - f_2(L,z_0) - 2x_{max}$.

1. For Direct rays: $f_1'(L,z_1) - x_1 = 0$.
2. For Reflected rays: $f_2'(L,z_1) - x_1 = 0$, for which $x_{max} = f_2(L,0) - f_2(L,z_0)$.
3. For Refracted rays: $f_2'(L,z_1) - x_1 = 0$, for which $x_{max} = f_2(L,z_{max}) - f_2(L,z_0)$.

Arrival times

$$t = \int dz \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \frac{n(z)}{c}$$

$$t(L,z) = \left(\frac{1}{c\,C\sqrt{n(z)^2 - L^2}}\right)$$
$$\left[n(z)^2 - L^2 + \left[Cz - \log\left(A\,n(z) - L^2 + \sqrt{A^2 - L^2}\sqrt{n(z)^2 - L^2}\right)\right]\frac{A^2\sqrt{n(z)^2 - L^2}}{\sqrt{A^2 - L^2}}\right.$$
$$\left. + A\sqrt{n(z)^2 - L^2}\log\left[n(z) + \sqrt{n(z)^2 - L^2}\right]\right]$$

1. For Direct rays: $\Delta t = t(L, z_0) - t(L, z_1)$
2. For Reflected rays: $\Delta t = t(L, z_0) - t(L, 0) + t(L, z_1) - t(L, 0)$
3. For Refracted rays: $\Delta t = t(L, z_0) - t(L, z_{max}) + t(L, z_1) - t(L, z_{max})$

Air Ray Tracing

- Same can be done for air ray tracing
- Should also get a shadow zone
- Not what we observe!
- Ignoring
  - Curved Earth
  - Ice-Air-Ice elevation deviation
  - Fresnel Effect near surface

Layer	Altitude Range (m)	A	B	C (m^{-1})
1	0 to 3217.48	1	0.000328911	0.000123309
2	3217.48 to 8363.54	1	0.000348817	0.000141571
3	8363.54 to 23141.80	1	0.000361006	0.000145679
4	23141.80 to 100000	1	0.000368118	0.000146522
5	$>$ 100000	1	0.000368117	0.000146522