Effect of Phase Center Offsets in LCP/RCP Correlation Maps

1 Defining H/V/LCP/RCP Waveforms

Waveform in the vertical polarization. The n index represents antenna number and the i index is for time bin.

$$v_n(t_i) = \sum_k e^{-j\omega_k t_i} V(\omega_k)\Delta\omega \tag{1}$$

Now horizontal polarization, giving all of the H-pol phase centers a time offset relative to the V-pol phase centers.

$$h_n(t_i) = \sum_k e^{-j(\omega_k t_i + \omega_k t_0)} H(\omega_k)\Delta\omega \tag{2}$$

$$r_n(t_i) = \frac{1}{\sqrt{2}}\left[v(t_i) + jh(t_i)\right] \tag{3}$$

$$\ell_n(t_i) = \frac{1}{\sqrt{2}}\left[v(t_i) - jh(t_i)\right] \tag{4}$$

Substituting Eqs. 7 and 8 into Eqs. 3 and 4:

$$r_n(t_i) = \frac{1}{\sqrt{2}}\sum_k \Delta\omega e^{-j\omega_k t_i}\left[V(\omega_k) + je^{-j\omega_k t_0} H(\omega_k)\right] \tag{5}$$

2 Cross-Correlations with LCP/RCP Waveforms

Now consider two antennas, and antenna 1 has a delay T with respect to 2. Then,

$$r_1(t_i) = \frac{1}{\sqrt{2}}\sum_k \Delta\omega e^{-j\omega_k t_i}\left[V(\omega_k) + je^{-j\omega_k t_0} H(\omega_k)\right] \tag{6}$$

And since

$$v_2(t_i) = \sum_k e^{-j\omega_k (t_i+T)} V(\omega_k)\Delta\omega \tag{7}$$

$$h_2(t_i) = \sum_k e^{-j[\omega_k (t_i+t_0+T)]} H(\omega_k)\Delta\omega \tag{8}$$

then

$$r_2(t_i) = \frac{1}{\sqrt{2}}\sum_k \Delta\omega e^{-j\omega_k (t_i+T)}\left[V(\omega_k) + je^{-j\omega_k t_0} H(\omega_k)\right] \tag{9}$$

Cross-correlating the RCP waveforms from antennas 1 and 2 (r_1 and r_2), and ignoring the normalization factor in the denominator for now, the get the following as a function of delay τ between the two RCP waveforms:

$$C_{12}^{rr}(\tau) = \sum_{k'} \Delta t \; r_1(t_i) r_2^*(t_i+\tau) \tag{10}$$

where the sum is over the region where the waveforms overlap for a given τ. Then substituting $r_1(t_i)$ and $r_2(t_i+\tau)$ into Eq. 15,

$$C_{12}^{rr}(\tau) = \frac{1}{2}\left[\sum_{k_1} \Delta\omega e^{-j\omega_{k_1} t_i}\left[V(\omega_{k_1}) + je^{-j\omega_{k_1} t_0} H(\omega_{k_1})\right]\right] \times \left[\sum_{k_2} \Delta\omega e^{+j\omega_{k_2}(t_i+T+\tau)}\left[V(\omega_{k_2}) + je^{-j\omega_{k_2} t_0} H(\omega_{k_2})\right]\right] \quad (11)$$

Collecting terms, we get:

$$C_{12}^{rr}(\tau) = \sum_{k_1}\sum_{k_2} (\Delta\omega)^2 e^{-j\left[\omega_{k_1} t_i - \omega_{k_2}(t_i+T+\tau)\right]} \times \left[V(\omega_{k_1})V^*(\omega_{k_2}) - je^{j\omega_{k_2} t_0} V(\omega_{k_1})H^*(\omega_{k_2}) + je^{-j\omega_{k_1} t_0} H(\omega_{k_1})V^*(\omega_{k_2}) + H(\omega_{k_1})H^*(\omega_{k_2})\right] \quad (12)$$

Likewise the LCP waveforms for antennas 1 and 2, where again antenna 1 has a delay T with respect to 2:

$$\ell_1(t_i) = \frac{1}{\sqrt{2}} \sum_k \Delta\omega e^{-j\omega_k t_i}\left[V(\omega_k) - je^{-j\omega_k t_0} H(\omega_k)\right] \quad (13)$$

$$\ell_2(t_i) = \frac{1}{\sqrt{2}} \sum_k \Delta\omega e^{-j\omega_k (t_i+T)}\left[V(\omega_k) - je^{-j\omega_k t_0} H(\omega_k)\right] \quad (14)$$

Then,

$$C_{12}^{\ell\ell}(\tau) = \sum_{k'} \Delta t \ \ell_1(t_i)\ell_2^*(t_i+\tau) \quad (15)$$

$$C_{12}^{\ell\ell}(\tau) = \sum_{k_1}\sum_{k_2} (\Delta\omega)^2 e^{-j\left[\omega_{k_1} t_i - \omega_{k_2}(t_i+T+\tau)\right]} \times \left[V(\omega_{k_1})V^*(\omega_{k_2}) + je^{j\omega_{k_2} t_0} V(\omega_{k_1})H^*(\omega_{k_2}) - je^{-j\omega_{k_1} t_0} H(\omega_{k_1})V^*(\omega_{k_2}) + H(\omega_{k_1})H^*(\omega_{k_2})\right] \quad (16)$$